AD-A264 229

(2)

UMENTATION PAGE

FINAL/01 MAR 90 TO 30 SEP 92

PARAMETRIC AND COMBINATORIAL PROBLEMS IN CONSTRAINED OPTIMIZATION

2304/DS

61102F

ROGER D. WETS

UNIVERSITY OF CALIFORNIA
OFFICE OF RESEARCH
DAVIS, CA 95616

AFOSR/NM
110 DUNCAN AVE, SUTE B115
BOLLING AFB DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

AFOSR-91-0050

11. SUPPLEMENTARY NOTES

12b. DISTRIBUTION CODE

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

13. ABSTRACT

The aggregation principle in stochastic optimization was exploited to build an algorithmic, progressive hedging, for solving such problems. Relationships were developed between asymptotic results for statistical estimators and those for the solution of stochastic optimization problems.

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL (UNLIMITED)

UNIVERSITY OF CALIFORNIA, DAVIS

BERKELEY • DAVIS • IRVINE • LOS ANGELES • RIVERSIDE • SAN DIEGO • SAN FRANCISCO SANTA BARBARA • SANTA CRUZ

DEPARTMENT OF MATHEMATICS DAVIS, CALIFORNIA 95616

Professor ROGER J-B WETS
Home: 2121 Pinehurst Ct., El Cerrito, CA 94530
Office: (916) 752-2936, home: (510) 232-0653
e-mail: rjbwets@ucdavis.edu — fax: (916) 752-6635

March 17, 1993

Dr. Neal Glassman
Air Force Office of Scientific Research
AFOSR/NM, Building 410
Bolling Air Force Base
Washington, D.C. 20332

Dear Neal:

Enclosed is my final report for the latest grant I held with the Air Force Office of Scientific Research. I am sorry to be somewhat late, but I had some health problems last month and because this quarter had a heavy teaching load, I couldn't quite get to the report as fast as I had wished for. Thanks for being patient.

Sincerely yours,

Roger

Roger J-B Wets

FINAL REPORT: Grant no. AFOSR-910050

STOCHASTIC DYNAMIC OPTIMIZATION: SOLUTION PROCEDURES AND APPROXIMATION SCHEMES

Roger J-B Wets

Department of Mathematics, University of California, Davis

March, 1993

PUBLICATION SUMMARY: October 1990 - December 1992

1. "Convergence of minima of integral functionals, with applications to optimal control and stochastic optimization," *Statistics and Decisions*, **11** (1993), to appear (with R. Lucchetti).
2. "Stability of stochastic programs with possibly discontinuous objective functions," *SIAM J. on Optimization*, submitted (with Z. Artstein).
3. "Laws of large numbers for random lsc functions," *Applied Stochastic Analysis & Stochastic Monographs*, **5** (1991), 101-120.
4. "Constrained estimation: consistency and asymptotics," *Applied Stochastic Models and Data Analysis*, **7** (1991), 17-32.
5. "Glivenko-Cantelli type theorems: an application of the convergence theory of stochastic suprema," *Annals Operations Research*, **30** (1991), 157-168 (with G. Salinetti).
6. "A dual strategy for the implementation of the aggregation principle in decision making under uncertainty," *Applied Stochastic Models and Data Analysis*, **8** accepted (with R.T. Rockafellar).
7. "Preprocessing in stochastic programming: the case of linear programs," *ORSA J. on Computing* **4** (1992), 45-59 (with S. Wallace).
8. "Preprocessing in stochastic programming: the case of capacitated networks," *ORSA J. on Computing*, submitted (with S. Wallace).
9. "The facets of the polyhedral set determined by the Gale-Hoffman inequalities," *Mathematical Programming*, accepted (with S. Wallace).

Work on Ph.D. Dissertations under partial support of this grant:

12. **D. Bradford,** *Analysis and Implementation of the Progressive Hedging Algorithm*, Graduate Group in Applied Mathematics, University of California, Davis, September 1993 (expected).
13. **M. Dong,** work started on robustness, and quantitative stability for (possibly constrained) estimation in mathematical statistics, Graduate Group in Applied Mathematics, University of California, Davis.

Computer routines available for distribution:

13. **M. Dong,** Stratification subroutines for multistage stochastic programming problems (1991).

DESCRIPTION OF THE RESULTS

In summary, I have authored or co-authored 9 papers, while two students have been working on their dissertations (occasionally supported as R.A.'s under this grant).

Numerical procedures for stochastic programming problems. The aggregation principle in stochastic optimization was exploited in "Scenarios and policy aggregation in optimization under uncertainty" (*Mathematics of Operations Research*, 16(1991), 119-147) to build an algorithmic procedure, known as the progressive hedging (PH) algorithm, for solving stochastic optimization problems. This method has received a number of experimental implementations and is starting to be relatively well understood. In [6] a dual strategy for the implementation of the aggregation principle is proposed. Bradford's dissertation [12] is mostly devoted to the questions raised by the implementation of the PH algorithm on machines with single or parallel processors. The analysis is both theoretical and numerical. In particular, he is able to obtain very sharp bounds on the optimal setting of the key parameters of the algorithm. Numerical experimentation with small to medium size problems confirms that a good choice of these parameters will result in significant speed up of the performance of the algorithm.

Dong [13] wrote subroutines that would facilitate the manipulation and machine feeding of the data associated with the formulation of a stochastic optimization problem.

In addition to solutions tools, it is useful for the modeler of a stochastic optimization problem to have at its disposal a number of numerical routines that would allow him to check if his problem is well-formulated. Because stochastic programming problems can be quite complex, modeling errors are potentially much more difficult to detect. Preprocessing tools that would identify infeasibilities, unboundedness, and the contribution of randomness, are developed for stochastic linear programs in [7], and further specialized to the case of problems whose recourse process can be formulated as capacitated network problems in [8]. These articles also report about the use of these techniques to analyze a number of stochastic programs found in the literature, and it is demonstrated that quite a bit of insight could be gained from their use. To obtain an efficient way of describing feasibility in capacitated network flow problems [8], there was a need to improve on the characterization of feasibility provided by the Gale-Hoffman inequalities, this led to [9] that proposes a very efficient (and elegant) procedure for eliminating the redundant inequalities.

Approximations for stochastic problems. Under this title, I shall review mainly applications of epi-convergence in a stochastic setting, in particular in the design of approximation schemes for stochastic optimization problems, to some estimation problems in mathematical statistics, and in stochastic homogenization.

The overall strategy for solving a stochastic optimization problem is to replace a given problem by a simpler one, often obtained by approximating the probability measure that governs the distribution of the random quantities. The approximating probability measure is usually obtained by some precise discretization scheme (usually requiring the calculation of conditional expectations) or by sampling. In both cases, one is faced with the question of knowing if the resulting problem will actually furnish an approximate solution.

When proceeding by discretization, one is confronted with the following question: given a random variable $\boldsymbol{\xi}$ with values in Ξ and probability distribution P, an integrand $f : \Xi \times X \to \overline{\mathbb{R}}$, and a sequence of integrands and probability measures $\{(f^\nu, P^\nu), \nu \in \mathbb{N}\}$ converging in some sense to (f, P), what can be said about the convergence of the functionals

$$(E^\nu f^\nu)(x) := \int_\Xi f^\nu(\xi, x) P^\nu(d\xi) \quad \text{to} \quad (Ef)(x) := \int_\Xi f(\xi, x) P(d\xi).$$

Our interest in epi-convergence comes from the need to claim that the minimizers of the functionals $E^\nu f$ converge to those of Ef. Articles [1] and [2] deal with this question. In the first one [1], the function f is fixed (doesn't depend on ν) with dom $f(\xi, \cdot)$ constant. The fact that weakly convergent measures $P^\nu \to P$ converge uniformly on certain classes of sets is used to obtain the epi- and pointwise convergence of the integral functionals $E^\nu f$ to Ef. The result is then applied to approximation schemes for deterministic and stochastic control problems. Although, this theorem improved on those previously known, it was still somewhat less than totally satisfactory. On one hand, we were getting more than desired (epi *and* pointwise convergence), and on the other hand, the assumption of constant domain was certainly less than wished for. The results of the next paper [2] no longer suffers from these shortcomings, and in addition allows for the dependence of f on ν. A new notion of convergence for integrands is proposed. There are no assumptions about the continuity of f with respect to ξ and this opens up a new range of potential applications. Stochastic optimization problems with discontinuous cost functions (a simple inventory example is analyzed in [2]), control of discrete events systems and stochastic integer programs involve integrands that are only exceptionally continuous in ξ.

The substitution of the given probability measure by an empirical measure generated by a sample is justified by the law of large numbers for random lsc functions proved in [3]. It is shown that if $\{f^\nu : \Xi \to \text{fcns}(X), \nu \in \mathbb{N}\}$ is a sequence of epi-iid random lsc functions, then $\text{e-lim}_\nu (1/\nu) \sum_1^\nu f^k = Ef^1$ almost surely (fcns(X) is the space of lsc functions defined on X). The result is proved [I] when X is finite dimensional.

The relationship between stochastic optimization and mathematical statistics is pursued in [4] and [5]. In [4], it is shown that the consistency of parametric or nonparametric estimators, even when there are constraints on the choice of the estimator, can be derived from the a.s.-convergence of the solutions of stochastic optimization problems described above. The relationship between the asymptotic results for statistical estimators and those for the solutions of stochastic optimization problems is also reviewed.

A standard approach to deriving the consistency of estimators relies on the almost sure uniform convergence of the empirical measures on certain classes of subsets of $\mathcal{A}$ (the σ-field of events). By introducing a certain topology, eventually called the *rough topology*, on the subspace of closed sets in $\mathcal{A}$, a criterion is given in [5] that can be used to check if a.s.-uniform convergence of the empirical measures will actually hold for any given class of sets. One of the implications of [5] is that we now know that a.s.-uniform convergence of the empirical measures holds for a much richer class of sets than those identified up to now.